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► To cite this version:

Randa Kassab, Frédéric Alexandre. A Heteroassociative Learning Model Robust to Interference. International Joint Conference on Computational Intelligence, Nov 2015, Lisboa, Portugal. hal-01232017

HAL Id: hal-01232017

<https://inria.hal.science/hal-01232017>

Submitted on 25 Nov 2015

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A Heteroassociative Learning Model Robust to Interference

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Abstract

Neuronal models of associative memories are recurrent networks able to learn quickly patterns as stable states of the network. Their main acknowledged weakness is related to catastrophic interference when too many or too close examples are stored. Based on biological data we have recently proposed a model resistant to some kinds of interferences related to heteroassociative learning. In this paper we report numerical experiments that highlight this robustness and demonstrate very good performances of memorization. We also discuss convergence of interests for such an adaptive mechanism for biological modeling and information processing in the domain of machine learning.

1 INTRODUCTION

Generalization is often reported as a desirable property of artificial neural networks. This phenomenon occurs if, when a network is presented with an example it has never seen before, it is able to interpolate a satisfactory response from the combination of close previously learned examples. Such a response can be judged satisfactory not only because from a limited learning phase the network behaves well in a wider domain but also because in some sense learning went beyond specific cases and was able to extract some general structures or regularities in the example space. In some cases, however, this property might be considered a flaw. This is the case for example when there is no useful topography in the example space or when the goal is to learn some arbitrary association. Consider for example learning to associate a phone number with a name: there is nothing to learn from the euclidean distance between two such numbers and you can in no way discover an association if it was not instructed to you before. This contrasts the cases of learning a general rule from a set of examples, as it is for example studied with layered architectures like the multilayer perceptron, versus learning by heart specific cases like in associative memories.

Neural models of associative memories have been proposed with recurrent networks like the Hopfield model (Hopfield, 1982) and the Willshaw model (Willshaw et al., 1969). Based on classical connectionist characteristics (like units with non linear activation functions and hebbian learning), the recurrent architecture of these networks indicates that learning is mainly focused on the inner characteristics of an example to be memorized and not on the elaboration of abstract representations in intermediate layers. Nevertheless, some problems can appear if too close examples are learned. In such a case, the network might elaborate an answer from the combination of several learned examples; what would be called generalization in other circumstances is called here interference.

As a consequence, models of associative memories are generally used as content addressable memories, where few prototypes are stored as stable states of the network and noisy or incomplete patterns are presented as inputs and reconstructed to the closest stored example. Beyond this use as an autoassociative memory (where initial input and final result have the same dimension), the adaptation to heteroassociative memory is straightforward: just virtually split the recurrent network in two sets of neurons A and B. The recurrent connectivity includes connections within A and within B (seen as two autoassociative memories) and between A and B (heteroassociative memory between the two sets of different dimension A and B). As configurations of A+B are learned as prototypes, proposing an incomplete pattern A (B neurons being set to 0) will result in the reconstruction of A+B, yielding the answer B. The main acknowledged weakness of these models is about their limited capacity of storage and the associated risk of catastrophic interference when this capacity is exceeded or when too close prototypes are stored (Graham and Willshaw, 1997; Knoblauch et al., 2010). The best solution to this problem is to require a sparse coding, which intrinsically also limits the maximum number of stored prototypes. An associated strategy is to orthogonalize the inputs and project their encoding in higher dimensions, which results in larger weight matrices to manipulate (McNaughton and Nadel, 1990).

Models of associative memory have also been studied for their complementarity with classical neural models of pattern matching like the multilayer perceptron and for the deep cognitive anchoring of this complementarity. Indeed, it was proposed 20 years ago (McClelland et al., 1995) that the brain exploits complementary learning systems, with a slow and procedural learning in the cortex, able to extract structures and regularities in the data and to generalize, compared with a quick learning of novel information in the hippocampus.

In a recent work, we have proposed a refinement of hippocampal model (Kassab and Alexandre, 2015), inspired from recent biological data (Samura et al., 2008). These data report heterogeneities in the hippocampal structure that might support the coexistence of autoassociative and heteroassociative memories in this region. Specifically, the hippocampus is a neuronal structure known to be involved in episodic memory (Tulving, 1972), corresponding to the storage of specific episodes including their context and their emotional or motivational significance. For example, the hippocampus is involved in contextual learning of pavlovian conditioning (Carrere and Alexandre, 2015), linking neutral stimuli and their context to biologically significant events (reward and punishment). Though primarily oriented toward biological modeling, we have also explained in (Kassab and Alexandre, 2015) the interest of such a segregation from an information processing point of view (cf. the concluding section for a summary). In addition, we have also postulated an additional mechanism for the association of autoassociative memories, that might result in a more robust system, particularly more resistant to interference. The goal of this paper is to evaluate more precisely the performances of this mechanism from an information processing point of view.

In the next section, we will present this model together with its formalism based on the associative memory initially proposed by Willshaw (Willshaw et al., 1969). Then we will report the experiments that were conducted to evaluate its resistance to interference and the associated results. We will conclude by explaining the interest of

such a mechanism both in neuroscience and in information processing domains.

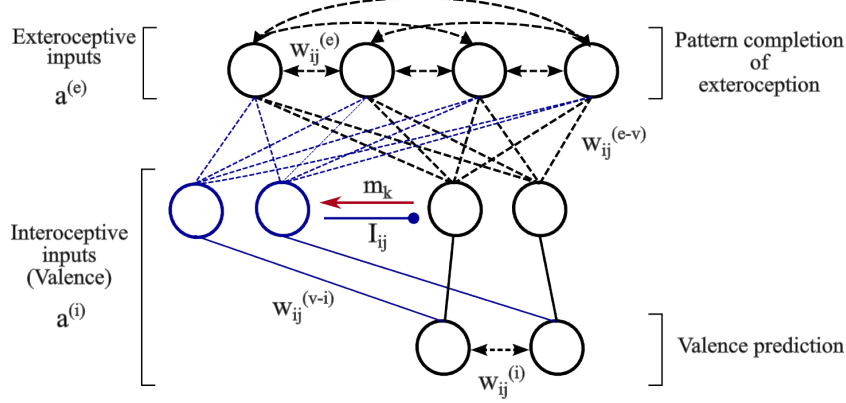


Figure 1. The architecture of the hippocampal model. Black lines denote the basic circuit of the model while blue lines denote changes in circuitry mediated by one group of associated cells (blue) following the detection of valence-overload interference (red arrow). Autoassociative and heteroassociative connectivities between hippocampal cells are denoted respectively by bidirectional dashed lines and simple dashed lines without arrows. Inhibitory connections between valence cells are denoted by lines ended with circles. Stable non-plastic connections, both excitatory and inhibitory, are denoted by solid lines.

2 MULTIPLE ASSOCIATIVE-MEMORY MODEL OF THE HIPPOCAMPUS

Our hippocampal model is made up of two autoassociative networks that are heteroassociatively linked through a layer of intermediate cells (Figure 1). The goal of this model is to store and recall specific episodes including a perceptual part (coming from the perception of the outer world: exteroception) and an emotional part (coming from the perception of internal cues of different valences related to pain and pleasure: interoception).

The two autoassociative networks considered in the model receive and store independently these two types of input patterns, $a^{(e)}$ and $a^{(i)}$. The layer of intermediate cells is organized into a small number of ordered groups of valence cells that receive valence-related information from the same interoceptive pathways as the interoceptive autoassociative network. The cells in the first group can be directly activated by interoceptive inputs to the model and can therefore be thought of as the primary valence cells. Interoceptive inputs on the cells in the other groups, which are termed associated cells, are conditional, that is, they can not evoke postsynaptic activity within associated cells unless a concomitant signal, m_k , related to the activity pattern of a precedent group is applied.

The valence cells belonging to the same group of intermediate cells are not interconnected but inhibitory connections, I_{ij} , exist between cells belonging to different groups. The inhibitory connections are not plastic. They are prewired such that an inhibitory connection from cell i to cell j exists ($I_{ij} = 1$) if the two cells belong respectively to different groups, k and l , and l precedes k ($l < k$). Thus, each group of associated cells, once activated, silences excitable cells in its preceding groups including the primary group of valence cells. This means that at most valence cells in one group can be active at a time.

The formation of extero-interoceptive associations is done at the level of heteroassociative links, $w_{ij}^{(e-v)}$, between the exteroceptive autoassociative network and the groups of intermediate valence cells. These latter provide direct excitatory input to the interoceptive autoassociative network through non-plastic connections, $w_{ij}^{(v-i)}$. These connections are prewired only between valence cells that are sensitive to the same kind of valence.

The classical binary version of the Willshaw network (Willshaw et al., 1969) is chosen as the basis for the implementation of both auto- and heteroassociative memory functions in the model. The neurons are simple McCulloch-

Pitts binary threshold units and learning begins with all the synaptic weights set to zero. Synaptic plasticity is achieved according to a clipped version of Hebbian learning: a single coincidence of presynaptic and postsynaptic activity changes the synaptic weight w_{ij} from 0 to 1, while further co-activations do not induce further changes. The recall process is done by presenting a cue pattern \tilde{x} and counting the dendritic sum for each cell j ($s_j = \sum_{i=1}^n w_{ij}\tilde{x}_i$) in one-time step. The output cells that have a dendritic sum equal to or higher than the number of active inputs are activated. The quality of a recalled pattern can be assessed according to its Hamming distance (HD) from the originally stored pattern (i.e. the number of elements that differ between the two patterns. For example, if $x=(0\ 1\ 1\ 1\ 0)$ and $y=(1\ 1\ 0\ 1\ 0)$ then $HD(x,y)=2$).

Similarly to cholinergic models of the hippocampus (Hasselmo et al., 1996; Meeter et al., 2004), our model operates in transition between two modes, storage and recall, depending on a hyperparameter ACh. This mechanism is inspired from biological data describing mode switching under the dynamic regulation of the levels of acetylcholine (ACh) released from septal cholinergic projections to the hippocampus. During recall, a retrieval cue, $a^{(e)}$, is applied to the exteroceptive autoassociative network. The pattern of activity obtained at the output, $\hat{a}^{(e)}$, drives retrieval in the heteroassociative network. An intermediate valence cell, l , can fire only if the dendritic sum of its excitatory inputs exceeds the threshold value and if it does not receive inhibitory inputs from other valence cells that have already fired. The activity of the intermediate valence cells, $\hat{a}^{(i)}$, triggers recall in the interoceptive autoassociative network yielding the valence prediction by the model, $\hat{a}^{(i)}$.

Just after delivery of the interoceptive information, two novelty-detection processes take place to compare the retrieved patterns to the actual patterns from extero- and interoception. The novelty condition occurs when the Hamming distance between two patterns exceeds pre-specified thresholds ($HD^{(e)} > e$ or $HD^{(i)} > v$). Novelty induces ACh dynamics that favor learning of new inputs, otherwise the model settles in recall mode.

During learning, excitatory intrinsic synaptic transmission along the recurrent connections is removed and activity in the model is purely driven by afferent extero- and interoceptive inputs, $a^{(e)}$ and $a^{(i)}$. In the model, two kinds of interference can occur due to a saturation, or overload of learning. The first kind of interference occurs within the autoassociative memories when too many or too close inputs are stored. It is called pattern overload and will not be considered here for simplicity (we will present sparse patterns during experiments to avoid this kind of interference). The second kind of interference is called valence overload and occurs when stimuli are simultaneously associated to different valences. Consider for example learning AB+, AC- and BD-, where A, B, C and D are exteroceptive stimuli and + and - are interoceptive valences. Since A and B are simultaneously associated to + and - valences, the recall of AB would probably generate an interference (both responses produced). The model deals with valence-overload interference by monitoring activity of intermediate valence cells, $y^{(v)}$. If any activity is observed among intermediate valence cells ($\sum_i y_i^{(v)} > 0$) in response to exteroceptive inputs a matching process takes place to determine whether this activity matches interoceptive valence-specific inputs. A mismatch ($HD^{(v)} > v$) signals a potential interference to a successive group of associated valence cells that become able to respond to valence-related inputs and rapidly silence valence cells that were active in preceding groups.

3 EXPERIMENTS

The validity of the proposed model is examined through a series of numerical experiments (cf. (Kassab and Alexandre, 2015) for the description of other numerical experiments with this model). The simulated model is configured with 150 cells in the exteroceptive autoassociative network and 3 cells in the interoceptive autoassociative network. The intermediate valence cells are organized into 5 groups of 3 cells each.

Inputs are provided to the model as two independent patterns of activity. The exteroceptive inputs are generated as random 150-element binary patterns with 6 elements being active (set to 1). The interoceptive inputs are modeled by 3 binary cells to differentiate positive, negative and neutral valence states. One of these cells switches to its active state according to whether a pleasant (100), unpleasant (010), or neutral (001) stimulus is present.

The performance is evaluated by comparing the output patterns recalled by the model against the original representation of the input patterns that were given as new information to be stored in the model. Specifically, two kinds of recall errors are considered when evaluating simulation results. Pattern completion errors which reflect the Hamming distance between the learned and retrieved activation for exteroceptive patterns, and valence prediction errors which reflect the Hamming distance between the correct and predicted valence. In both cases, errors are scored when Hamming distance is greater than zero.

Two types of simulations are set out to test the model for its ability to rapidly link exteroceptive patterns and

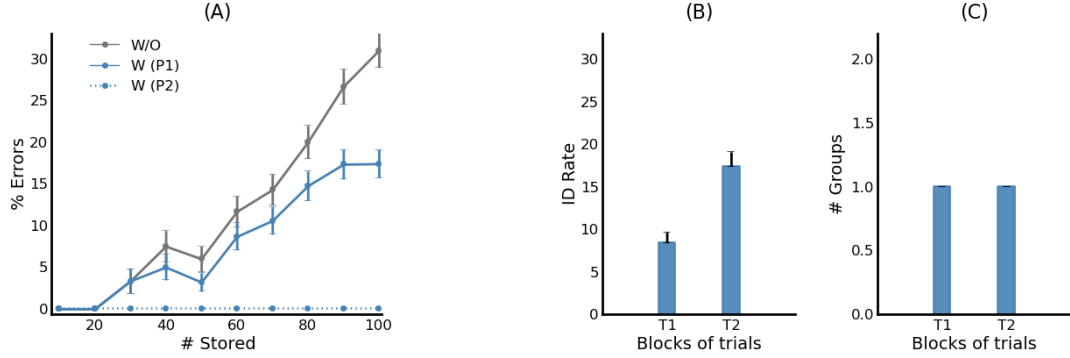


Figure 2. Influence of the number of stored patterns on the accuracy of valence prediction. (A) Percentage of prediction errors of the model without associated cells (w/o) and with associated cells after one block (W (P1)) and two blocks (W (P2)) of training trials. (B,C) Details of the simulation results when the block size is set to 100. B: Rates of interference detection during the first (T1) and second (T2) training trials. C: Number of groups of associated cells needed to resolve interference detected during training trials T1 and T2.

their emotional valences while avoiding valence overload interference. The first set of simulations examines the effect of the number of stored patterns on the accuracy of valence prediction. The model is tested under full-cue and partial-cue recall conditions. In both cases the performance of the proposed model is compared with that of a reduced model with the groups of associated cells removed. The second set of simulations focuses on how to quantify the ability of associated valence units to orthogonalize conflicting associations arising from a change in previously learned valence values. In all simulations, we assume that the representations of input patterns are sufficiently pulled apart so that no interference can occur at the level of autoassociative memories (no pattern overload). This is important to ensure that the effects observed in the model stem directly from valence overload interference at the level of heteroassociative links between exteroceptive and interoceptive patterns. All results are averaged over 5 simulation runs and are displayed throughout the figures as mean \pm standard error of the mean. The novelty-detection thresholds, e and v , are set to zero for all the simulations.

4 RESULTS

4.1 Overloading

The first set of simulations is run by varying the number of training patterns and observing how valence prediction is affected with and without the groups of associated cells included in the model (Figure 2). Training patterns are presented randomly into blocks of N trials. Following the first presentation of training patterns, the prediction of the full and reduced models is perfect up to $N=20$, after which point prediction errors begin to occur more frequently with increasing size of the blocks of training trials. At $N=100$, the percentage of prediction errors is a little more than 30% for the reduced model but falls to about 17% for the full model. This reduction results from the identification of about 8% of the stored associations as interfering associations (Figure 2B). Interference effect is accordingly reduced through the recruitment of one group of associated cells (Figure 2C). During the second presentation of training patterns, the full model detects all the interfering associations that remain and orthogonalizes them using the same group of associated cells (Figure 2C). Therefore, the performance of valence prediction differs significantly between the two models after the second presentation of training patterns: the reduced model continues to commit the same prediction errors while the proposed model performs with no errors at all.

Next, the model is trained in the same manner as in the previous simulations except that recall is triggered by partial versions of the original trained patterns. Specifically, the block size is set to 100 training patterns and the model is cued with partial versions with either 1, 2 or 3 of the 6 active inputs turned off. Figures 3C and D show that the accuracy of valence prediction with the 1/6 partial-cue condition is the same as that obtained with the full-cue condition. This is because exteroceptive patterns are almost perfectly reconstructed as shown in Figures 3A and

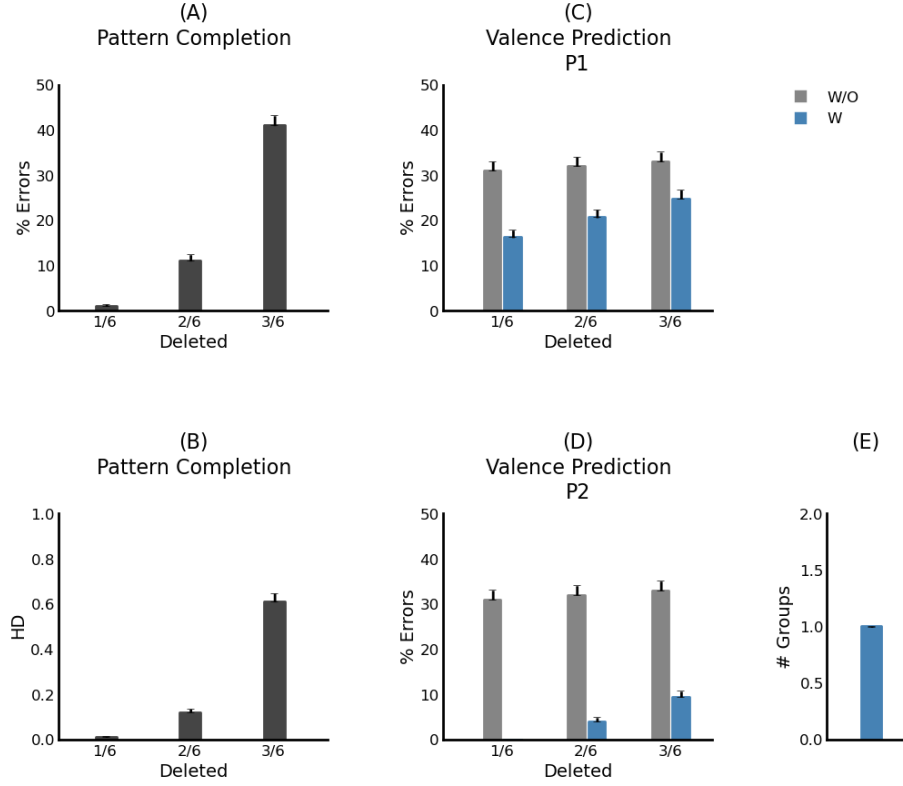


Figure 3. Performance of the proposed model after training on 100 input patterns. The model is tested using partial cues in which 1, 2, or 3 out of 6 active elements in the original inputs are turned off. (A) Pattern completion performance, defined as the percentage of retrieved patterns that differ at least by one element from the originally stored patterns. (B) Pattern completion performance, defined in terms of Hamming distance between the stored and retrieved patterns. (C, D) Valence prediction performance of the proposed model with (w) and without (w/o) associated cells after one and two blocks of training trials. (E) Maximal number of groups of associated cells needed to resolve interference detected under all simulation conditions (one and two blocks of training trials P1 and P2, and for 1/6, 2/6 and 3/6 partial-cue conditions).

B. The removal of two or three of the six active cues causes a proportional decrease in the accuracy of pattern completion of exteroceptive patterns. Consequently, the improvement in valence prediction by the proposed model is less pronounced but still highly significant as compared to the reduced model. For all percentages of removal tested, the model makes use of one group of associated cells to tackle valence-overload interference (Figure 3E).

4.2 Discrimination and reversal

Here we investigate the functional significance of the groups of associated cells using numerical simulations with reversal learning tasks. The task in the first set of simulations involves two phases. In the first phase the model is presented repeatedly with 50 training patterns [e.g. A+, B-, C (neutral), etc.] over 4 blocks of trials and the percentage of prediction errors made at the beginning of each trial is measured and displayed in Figure 4A. This is a simple discrimination learning problem similar to those tested in the previous simulations. Thus as was observed before, valence-overload interference occurs at the early stages of learning and exhibits the recruitment of one group of associated cells to tackle it. When the groups of associated cells are removed the reduced model shows impaired performance that persists over the repeated trials. In the second phase, emotional valences of the training patterns are randomly changed to other value with a probability of 50% [e.g. A-, B (neutral), C (neutral), etc.]. As shown

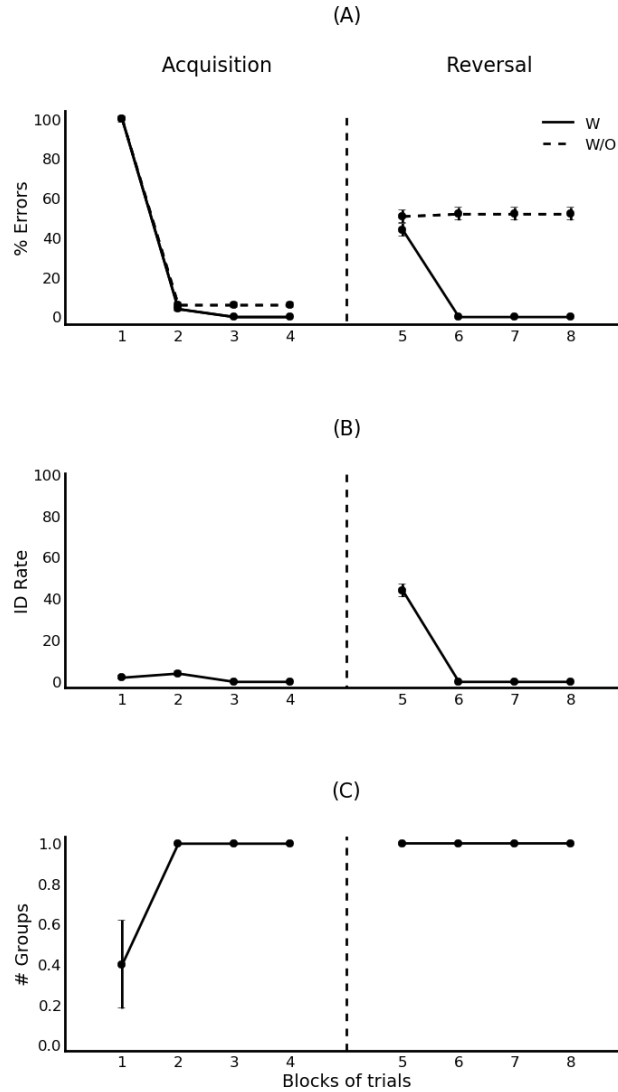


Figure 4. Discrimination reversal learning. (A) Percentage of prediction errors of the model with (w) and without (w/o) associated cells. (B) Rates of interference detection over each block of trials. (C) Number of groups of associated cells needed to resolve interference across the different blocks of trials.

in Figure 4A the proposed model quickly learns to reverse its behavior as all the emotionally changed patterns are detected and learned on the first training trials after reversal. On the other hand, the reduced model fails to acquire the new associations since the old ones have not been unlearned.

The second set of simulations involves a cue-context reversal learning task similar to that established by (Levy-Gigi et al., 2011) to investigate reversal learning in patients with mild amnesic cognitive impairment. To simulate this task, three groups of 4 exteroceptive patterns each are formed such that one of the 6 active elements is used to encode the presence of a sensory cue and the others to encode contextual cues. No overlap is allowed between cells encoding for different cues or contexts (cf. Table 1).

In the first phase of acquisition, the model is repeatedly presented with the training patterns in the first group and valence prediction is evaluated over four blocks of training trials. Figure 5 shows that both full and reduced models make correct valence prediction after a single exposure to the training patterns. Then, the reversal phase is

Table 1: The experimental design of the task of (Levy-Gigi et al., 2011). Note. A–H refer to eight cue shapes, 1–8, eight contexts, + and – indicate respectively positive and negative valences.

Training patterns			Task	
Group 1 (original)	Group 2 (cue reversal)	Group 3 (context reversal)	Phase 1 (acquisition)	Phase 2 (retention & reversal)
A1+	E1–	A5–	Group1	Group1
B2+	F2–	B6–		Group2
C3–	G3+	C7+		Group3
D4–	H4+	D8+		

immediately followed by exposing the models to new training patterns from the second and third groups, in addition to the old ones. The training patterns are also presented repeatedly four times in random order. The results show that, in the first block of trials, valence prediction errors are made for both new and old patterns. This reflects the fact that heteroassociative connections are irrelevantly strengthened between the original patterns and valences of new patterns. When interference is detected, one group of valence-associated cells is recruited and prediction errors fall to zero rapidly on the third block of trials after reversal. In contrast, the number of prediction errors the reduced model makes is still the same as the blocks progress for the same reason stated above.

5 DISCUSSION

This paper sets emphasis on a class of connectionist models, associative memories, with powerful properties for learning by heart specific patterns and recalling them from partial information. Such models can be simply used for pattern retrieval but also in heteroassociation between two classes of inputs. In our biologically informed model (Kassab and Alexandre, 2015), we propose such a heteroassociation between exteroception and interoception. From an information processing point of view, we explain in that paper that a heteroassociation between two data spaces of different size leads to more robust retrieval than a simple autoassociation with a flat vector concatenating both kinds of information because the evaluation of the Hamming distance between stored and actual patterns would consider in this latter case that one error in any dimension yields the same penalty, which is obviously not the case. Beyond the case for pavlovian conditioning with intero- and exteroceptive cues, we believe that it is not rare in the information processing domain to cope with such associations between data of different dimensions, as it is the case for example with labeled data (high-dimensional data associated with a symbolic label). In this case, we claim that combining auto- and heteroassociation as proposed here results in more robustness in the retrieval phase.

Results reported in the present paper were centered on another powerful property of our model, for managing interferences. When an association is learned between a high dimensional data space and a smaller space representing labels (valences in the present case), one central problem is about the association of close patterns with different labels or of different combinations of patterns with different labels. This classical problem has been termed configural learning (Buhusi and Schmajuk, 1996). Based on biological data and also benefiting from the separation between exteroceptive and interoceptive data, we have proposed in the present model a mechanism able to automatically detect such interference at the heteroassociative level and to trigger new learning accordingly. The experiments reported here show that our model is very efficient at performing such a learning. In addition, this learning process is very quick, which preserves another important specificity of episodic learning.

These results have been described here in the framework of information processing, but one of the experiments has also been designed to reproduce behavioral and cognitive data in the medical domain for amnesic impairments (Levy-Gigi et al., 2011). Related medical data strongly suggest the central role of the hippocampus in this memory process, giving additional interest to the complementary learning system hypothesis (O'Reilly et al., 2011). This cognitive framework also postulates how procedural learning in the cortex, slowly learning and able of generalization, might be instructed by specific cases learned quickly in the hippocampus avoiding interferences. Coming back to the information processing domain, we believe that better understanding these processes and modeling them more faithfully is of high interest for designing machine learning systems combining different memory processes

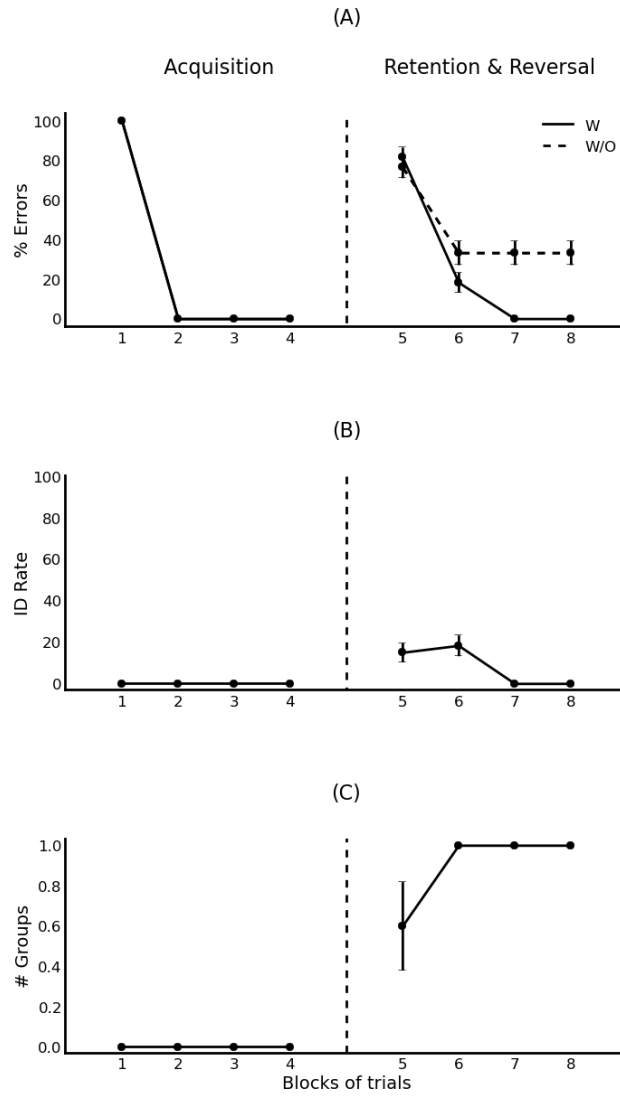


Figure 5. Cue-context reversal learning. (A) Percentage of prediction errors of the model with (w) and without (w/o) associated cells. (B) Rates of interference detection over each block of trials. (C) Number of groups of associated cells needed to resolve interference across the different blocks of trials.

for higher performances.

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